Comment on "Universality of the 1/3 shot-noise suppression factor in nondegenerate diffusive conductors"

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We argue that the nearly 1/3 suppression of shot noise in nondegenerate diffusive contacts recently obtained by Gonzalez et al. is due to the specific choice of the energy-independent elastic scattering time.

Recently, González *et al.* [1] performed Monte Carlo simulations of the shot noise in nondegenerate diffusive conductors. They obtained that the noise was 1/3 of the full Poisson value $S_I = 2eI$ for the 3D electron spectrum and 1/2 of it for the 2D one within the calculational error and attributed this suppression to Coulomb repulsion of electrons. More recently, Beenakker [2] performed analytical calculations of noise for the same model and obtained values slightly different from 1/3 and 1/2. However the analytical solution for this case could be obtained only by omitting the diffusion term in the drift-diffusion equation, which may be rigorously justified only for an infinite-dimensional system. All this could lead one to thinking that the 1/3 shot-noise reduction, which results from the Pauli principle in degenerate Fermi systems and is shown to be universal for them [3,4], may be a more general effect [5].

We show that unlike the 1/3 noise reduction in degenerate systems, the noise suppression is *nonuniversal* in nondegenerate systems even for a given dimensionality and that Coulomb interaction does not always suppress shot noise.

Unlike the degenerate systems whose kinetics is determined by the scattering time at the Fermi surface, nondegenerate systems should be sensitive to its energy dependence since all electrons in the conduction band contribute to transport and noise. We argue that the nearly 1/3 suppression of shot noise obtained by González *et al.* is due to the particular choice of the energy-independent elastic scattering time. Below we present calculations for a specific case where exact analytical results are available and the shot noise is not suppressed at all.

Consider a long and narrow 3D semiconductor microbridge connecting two massive electrodes of the same material. Suppose that the temperature is low enough for the electrons in the conduction band to be nondegenerate yet their concentration n_0 is sufficient for the screening length $\lambda = (4\pi e^2 n_0/T)^{-1/2}$ to be much smaller than the dimensions of the contact.

Starting from the standard Boltzmann-Langevin equation

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial}{\partial \mathbf{p}}\right]\delta f + \delta I = -e\,\delta\mathbf{E}\frac{\partial f}{\partial \mathbf{p}} + \delta J^{ext} \quad (1)$$

and introducing the energy variable $\epsilon = p^2/2m + e\phi(\mathbf{r})$, one may present the quasistatic local fluctuation of current density in the form

$$\delta\mathbf{j}(\mathbf{r}) = -e\int_{e\phi(\mathbf{r})}^{\infty} d\epsilon\,\delta\left[N(\epsilon - e\phi)D(\epsilon - e\phi)\nabla f(\epsilon, \mathbf{r}, t)\right] + \delta\mathbf{j}^{ext}(\mathbf{r}, t), \quad (2)$$

where N and D are energy-dependent density of states and diffusion coefficient, and $f(\epsilon)$ is the symmetric part of distribution function in the momentum space [6]. Note that the term containing fluctuations of self-consistent electric field is absorbed into the gradient term by virtue of the definition of ϵ. The correlation function of extraneous currents $\delta\mathbf{j}^{ext}$ may be presented in the form

$$\left\langle \delta j^{ext}_\alpha(\mathbf{r})\delta j^{ext}_\beta(\mathbf{r}')\right\rangle_\omega = 4e^2\delta_{\alpha\beta}\delta(\mathbf{r} - \mathbf{r}') \times \int d\epsilon\,N(\epsilon - e\phi)D(\epsilon - e\phi)f(\epsilon, \mathbf{r})[1 - f(\epsilon, \mathbf{r})]. \quad (3)$$

In principle, one has first to determine fluctuations of potential ϕ from the self-consistency equation to calculate fluctuations of distribution function and the current. However it is possible to choose the energy dependence of the elastic scattering time τ in such a way that quasistationary fluctuations of distribution function $f(\epsilon)$ may be determined independently of $\delta\phi$. Specify its energy dependence in the form $\tau(\epsilon) \propto \epsilon^{-3/2}$. An energy dependence of this type, though unusual, is not forbidden by any fundamental laws and does not result in a divergency of measurable quantities. In this case, the product ND is energy-independent, and the fluctuation of the current through the contact is obtained by averaging (2) over the contact volume. The gradient term in (2) is eliminated by integration over the longitudinal coordinate x, and provided that $f \ll 1$, the spectral density of current noise takes the form

$$S_I = \frac{4e^2A}{L^2}\int dx\int_0^\infty d\epsilon\,D(\epsilon - e\phi)N(\epsilon - e\phi)f(\epsilon, x), \quad (4)$$

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where L and A are the contact length and cross-section area [6].

In the absence of inelastic collisions, the average distribution function $f(\epsilon, x)$ may be determined from the condition of particle-current conservation at a given energy,

$$\frac{\partial}{\partial x}\left[N(\epsilon - e\phi)D(\epsilon - e\phi)\frac{\partial f}{\partial x}\right] = 0. \tag{5}$$

At the left edge of the contact, the boundary condition for this diffusion equation is

$$f(\epsilon, 0) = f_0\theta(\epsilon)\ \exp(-\epsilon/T), \tag{6}$$

where f_0 is a small temperature-dependent prefactor. The distribution function at the right end of the contact is shifted by eV with respect to $f(\epsilon, 0)$:

$$f(\epsilon, L) = f_0\theta(\epsilon + |eV|)\exp(-\epsilon/T - |eV|/T). \tag{7}$$

As $|eV| \gg T$, it may be neglected at $\epsilon > 0$. As the product ND is constant, the solution of (5) is

$$f(\epsilon, x) = (1 - x/L)f(\epsilon, 0). \tag{8}$$

This expression suggests that the current is comprised of electrons injected into the contact from the left electrode that retain their total energy in process of diffusion. The electrons from the right electrode do not contribute to the current because thay cannot overcome the potential drop across the contact. Hence one obtains from (4) that $S_I = (2e^2AT/L)DNf_0$ and $I = (eAT/L)DNf_0$. In other words, $S_I = 2eI$ and not $2eI/3$ despite the Coulomb interaction. This situation has much in common with the case of normal metals, where the $1/3$ suppression of shot noise is entirely due to the Pauli principle and Coulomb interaction does not result in its additional suppression [6]. Note that since in both cases the product ND is constant, fluctuations of $f(\epsilon)$ (but not those of $f(\mathbf{p})$!) are not affected by fluctuations of self-consistent electric field.

In actual semiconductors, $\tau(\epsilon)$ is constant only in a limited range of energies in the case of neutral impurities. In the most typical case of ionized-impurity scattering, the Brooks - Herring formula gives $\tau \propto \epsilon^{3/2}$. Since both types of impurities can be present in the same sample, one should expect a scatter of experimental shot-noise suppression factors.

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[1] T. González, C. González, J. Mateos, D. Pardo, L. Reggiani, O.M. Bulashenko, and J. M. Rubí, Phys. Rev. Lett. **80**, 2901 (1998).

[2] C. W. J. Beenakker, cond-mat/9810117.

[3] Yu. V. Nazarov, Phys. Rev. Lett. **73**, 134 (1994).

[4] E. V. Sukhorukov and D. Loss, Phys. Rev. Lett. **80**, 4959 (1998).

[5] This point was disputed by R. Lanadauer, Nature **392**, 658 (1998).

[6] K. E. Nagaev, Phys. Lett. A **169**, 103 (1992).